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Confinement in the Abelian-Higgs–type theories: string picture and field correlators*

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Abstract

Field correlators and the string representation are used as two complementary approaches for the description of confinement in the SU(N)-inspired dual Abelian-Higgs–type model. In the London limit of the simplest, SU(2)-inspired, model, bilocal electric field-strength correlators have been derived with accounting for the contributions to these averages produced by closed dual strings. The Debye screening in the plasma of such strings yields a novel long-range interaction between points lying on the contour of the Wilson loop. This interaction generates a Lüscher-type term, even when one restrics oneself to the minimal surface, as it is usually done in the bilocal approximation to the stochastic vacuum model. Beyond the London limit, it has been shown that a modified interaction appears, which becomes reduced to the standard Yukawa one in the London limit. Finally, a string representation of the SU(N)-inspired model with the Θ-term, in the London limit, can be constructed.

INTRODUCTION

The Stochastic Vacuum Model (SVM) [1] is nowadays commonly recognized as a promising nonperturbative approach to QCD (see Ref. [2] for reviews). Within the so-called bilocal or Gaussian approximation, well confirmed by the existing lattice data [3, 4], this model is fully described by the irreducible bilocal gauge-invariant field strength correlator (cumulant), $\langle\langle F_{\mu\nu}(x)\Phi(x,x')F_{\lambda\rho}(x')\Phi(x',x)\rangle\rangle$. Here, $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu - ig[A_\mu, A_\nu]$ stands for the Yang-Mills field-strength tensor, $\Phi(x,y) \equiv \frac{1}{N_c}\mathcal{P}\exp\left(ig\int\limits_y^x A_\mu(u)du_\mu\right)$ is a parallel transporter factor along the straight-line path, and $\langle\langle\mathcal{O}\mathcal{O}'\rangle\rangle \equiv \langle\mathcal{O}\mathcal{O}'\rangle - \langle\mathcal{O}\rangle\langle\mathcal{O}'\rangle$ with the average defined *w.r.t.* the Euclidean Yang-Mills action. It is further convenient to parametrize the bilocal cumulant by the two coefficient functions D and D_1 [1, 2] as follows:

$$\frac{g^2}{2}\langle\langle F_{\mu\nu}(x)\Phi(x,x')F_{\lambda\rho}(x')\Phi(x',x)\rangle\rangle = \hat{1}_{N_c\times N_c}\Big\{(\delta_{\mu\lambda}\delta_{\nu\rho} - \delta_{\mu\rho}\delta_{\nu\lambda})D\left((x-x')^2\right) +$$

$$+\frac{1}{2}\left[\partial_\mu^x((x-x')_\lambda\delta_{\nu\rho} - (x-x')_\rho\delta_{\nu\lambda}) + \partial_\nu^x((x-x')_\rho\delta_{\mu\lambda} - (x-x')_\lambda\delta_{\mu\rho})\right]D_1\left((x-x')^2\right)\Big\}. \tag{1}$$

After that, setting for the nonperturbative parts of the D- and D_1-function various *Ansätze*, one can apply SVM to calculations of the high-energy scattering processes [5] or test these *Ansätze* in the lattice experiments [3, 4]. However, from the pure field-theoretical point of view, a challenge remains to derive the coefficient functions analytically. Unfortunately, in QCD, this problem looks too complicated.

To proceed with, it is therefore reasonable to derive field-strength correlators not in QCD itself, but rather in some Abelian-type QCD-inspired models, which inherit confinement and allow for its analytic description. These include SU(2)- [6] and SU(3)- [7] inspired dual Abelian-Higgs–type theories, as well as 3D compact QED [8]. The bilocal field-strength cumulant in these theories has been studied in Refs. [9, 10, 11], respectively. In the present minireview, we will briefly survey the results which concern the dual Abelian-Higgs–type theories, as well as their further elaborations performed in Ref. [12]. For the sake of simplicity, we will restrict ourselves to the SU(2)-inspired case, *i.e.*, a simple dual Abelian-Higgs model (DAHM), although the SU(3)-generalization is straightforward [10].

One important fact for the further discussion is that in DAHM a sector with closed dual strings [13] exists. Such closed strings are short-living (virtual) objects, whose typical sizes are much smaller than the typical distances between them. This means that, similarly to monopoles in 3D compact QED, closed strings can be treated in the dilute-plasma approximation. Moreover, in the leading (semi-classical) approximation, the interaction of closed dual strings with large open ones, which end up at external quarks, can be disregarded at all. This is precisely the approximation in which field-strength correlators have been evaluated in Refs. [9, 10]. A leading correction to these semi-classical expressions, which stems from the interaction of closed strings with the open ones, has been found in Ref. [12] and will be reviewed below.

The outline of the minireview is as follows. In the next Section, we will first mention a correspondence, based on the Abelian-projection method, between the DAHM and the SU(2)-QCD, which will be needed for the future purposes. Secondly, we will briefly review the main results of a calculation of electric field-strength correlators in the approximation when closed strings are disregarded. In the subsequent Section, after a brief review of properties of the grand canonical ensemble of closed strings, we will consider the contribution of these objects to the field-strength correlators. In the same Section, we will also discuss two types of corrections to the $\bar{q}q$-potential - due to closed strings and due to the deviation from the London limit. In the last Section, we will present a string representation of the SU(N)-inspired analogue of DAHM extended by the Θ-term. The main results will finally be quoted in Summary.

ELECTRIC FIELD-STRENGTH CORRELATORS IN THE ABSENCE OF CLOSED STRINGS

The model

To derive from the Lagrangian of the SU(2)-gluodynamics an IR effective theory, based on the assumption of condensation of Abelian-projected monopoles, one usually employs the so-called Abelian dominance hypothesis [14]. It states that the off-diagonal (in the sense of the Cartan decomposition) fields can be disregarded, since after the Abelian projection those can be shown to become very heavy and therefore irrelevant to the IR region. The

action describing the remaining diagonal fields and Abelian-projected monopoles reads

$$S_{\text{eff.}}\left[a_\mu, f^{\rm m}_{\mu\nu}\right] = \frac14 \int d^4x \left(f_{\mu\nu} + f^{\rm m}_{\mu\nu}\right)^2. \tag{2}$$

Here, $a_\mu \equiv A^3_\mu$, $f_{\mu\nu} = \partial_\mu a_\nu - \partial_\nu a_\mu$, and the monopole field-strength tensor $f^{\rm m}_{\mu\nu}$ obeys Bianchi identities modified by monopoles, $\partial_\mu \tilde f^{\rm m}_{\mu\nu} \equiv \frac12 \varepsilon_{\mu\nu\lambda\rho}\partial_\mu f^{\rm m}_{\lambda\rho} = j^{\rm m}_\nu$. The monopole currents $j^{\rm m}_\mu$'s should eventually be averaged over in the sense, which will be specified below.

To proceed with the investigation of the monopole ensemble, it is useful to dualize the theory under study. This yields the following expression for the partition function:

$$\mathcal{Z} = \left\langle \int \mathcal{D}B_\mu \exp\left[-\int d^4x \left(\frac14 F^2_{\mu\nu} - iB_\mu j^{\rm m}_\mu\right)\right]\right\rangle_{j^{\rm m}_\mu}, \tag{3}$$

where B_μ is the magnetic vector-potential dual to the electric one, a_μ, and $F_{\mu\nu} = \partial_\mu B_\nu - \partial_\nu B_\mu$. Once the $j^{\rm m}_\mu$-dependence of the action became explicit, it is now possible to set up the properties of the monopole ensemble. To describe the condensation of monopoles, it is first necessary to specify $j^{\rm m}_\mu$ as the collective current of N of those: $j^{{\rm m}\,(N)}_\mu(x) = g_m \sum\limits_{n=1}^{N} \oint dx^n_\mu(s)\delta(x - x^n(s))$. Here, the world line of the n-th monopole is parametrized by the vector $x^n_\mu(s)$, and g_m is the magnetic coupling constant, related to the QCD coupling constant g via the quantization condition $gg_m = 4\pi n$ with n being an integer. In what follows, we will for concreteness restrict ourselves to the monopoles possessing the minimal charge, *i.e.* set $n = 1$, although the generalization to an arbitrary n is straightforward. Further, it is necessary to set for the measure $\langle \ldots \rangle_{j^{\rm m}_\mu}$ the following expression [15]:

$$\left\langle \exp\left(i\int d^4x B_\mu j^{\rm m}_\mu\right)\right\rangle_{j^{\rm m}_\mu} = 1 + \sum_{N=1}^{\infty} \frac{1}{N!}\left[\prod_{n=1}^{N} \int\limits_0^\infty \frac{ds_n}{s_n} {\rm e}^{2\lambda\eta^2 s_n} \int\limits_{u(0)=u(s_n)} \mathcal{D}u(s'_n)\right] \times$$

$$\times \exp\left\{\sum_{l=1}^{N} \int\limits_0^{s_l} ds'_l \left[-\frac14 \dot u^2(s'_l) + ig_m \dot u_\mu(s'_l) B_\mu(u(s'_l))\right] - \lambda \sum_{l,k=1}^{N} \int\limits_0^{s_l} ds'_l \int\limits_0^{s_k} ds''_k \delta\left[u(s'_l) - u(s''_k)\right]\right\}. \tag{4}$$

Here, the vector $u_\mu(s'_n)$ parametrizes the same contour as the vector $x^n_\mu(s)$. Clearly, the world-line action standing in the exponent on the R.H.S. of Eq. (4) contains besides the usual free part also the term responsible for the short-range repulsion (else called self-avoidance)

of the trajectories of monopoles. Equation (4) can further be rewritten as an integral over the dual Higgs field as follows:

$$\left\langle \exp\left(i\int d^4x B_\mu j_\mu^{\rm m}\right)\right\rangle_{j_\mu^{\rm m}} = \int \mathcal{D}\Phi\mathcal{D}\Phi^* \exp\left\{-\int d^4x\left[|D_\mu\Phi|^2+\lambda\left(|\Phi|^2-\eta^2\right)^2\right]\right\}, \qquad (5)$$

where $D_\mu = \partial_\mu - ig_m B_\mu$ is the covariant derivative. Finally, substituting Eq. (5) into Eq. (3), we arrive at the DAHM:

$$\mathcal{Z} = \int |\Phi|\,\mathcal{D}\,|\Phi|\,\mathcal{D}\theta\mathcal{D}B_\mu \exp\left\{-\int d^4x\left[\frac{1}{4}F_{\mu\nu}+|D_\mu\Phi|^2+\lambda\left(|\Phi|^2-\eta^2\right)^2\right]\right\}, \qquad (6)$$

where $\Phi(x) = |\Phi(x)|\,{\rm e}^{i\theta(x)}$. The masses of the dual vector boson and of the dual Higgs field, derivable upon the substitution $\Phi(x) = \eta + \frac{\varphi(x)}{\sqrt{2}}$, read $m_B \equiv m = \sqrt{2}g_m\eta$ and $m_H = 2\eta\sqrt{\lambda}$, respectively. Clearly, the two main assumptions, made in course of this derivation, were the neglection of the off-diagonal degrees of freedom and the postulate that the monopole condensate can be modeled by the dual Higgs field.

Bilocal electric field-strength correlator

In order to investigate the bilocal cumulant of electric field strengths in the model (6), it is necessary to extend this model by external electrically charged test particles [*i.e.* particles, charged *w.r.t.* the Cartan subgroup of the original SU(2)-group]. It is therefore natural to call these particles simply "quarks". Such an extension can be performed by adding to the action (2) the term $i\int d^4x a_\mu j_\mu^{\rm e}$ with $j_\mu^{\rm e}(x) \equiv g\oint\limits_C dx_\mu(s)\delta(x-x(s))$ standing for the conserved electric current of a quark, which moves along a certain closed contour C. Then, performing the dualization of the so-extended action and summing up over monopole currents according to Eq. (4), we arrive at Eq. (6) with $F_{\mu\nu}$ replaced by $F_{\mu\nu}+F_{\mu\nu}^{\rm e}$. Here, $F_{\mu\nu}^{\rm e}$ stands for the field-strength tensor generated by quarks according to the equation $\partial_\mu\tilde{F}_{\mu\nu}^{\rm e} = j_\nu^{\rm e}$. A solution to this equation reads $F_{\mu\nu}^{\rm e} = -g\tilde{\Sigma}_{\mu\nu}^{\rm e}$, where $\Sigma_{\mu\nu}^{\rm e}(x) \equiv \int\limits_{\Sigma^{\rm e}} d\sigma_{\mu\nu}(\bar{x}(\xi))\delta(x-\bar{x}(\xi))$ is the so-called vorticity tensor current defined at an arbitrary surface $\Sigma^{\rm e}$ (which is just the world sheet of an open dual Nielsen-Olesen string), bounded by the contour C, and ξ is a 2D-coordinate.

From now on, we will be interested in the London limit of DAHM, $\lambda\to\infty$, where it admits an exact string representation. In that limit, the partition function (6) with external quarks reads

$$\mathcal{Z} = \int \mathcal{D}B_\mu \mathcal{D}\theta \exp\left\{-\int d^4x \left[\frac{1}{4}\left(F_{\mu\nu} + F^{\rm e}_{\mu\nu}\right)^2 + \eta^2 \left(\partial_\mu\theta - g_m B_\mu\right)^2\right]\right\}. \tag{7}$$

In Eq. (7), one next performs a decomposition of the phase of the dual Higgs field $\theta = \theta^{\rm sing.} + \theta^{\rm reg.}$, where the multivalued field $\theta^{\rm sing.}(x)$ describes a certain configuration of dual strings and obeys the equation [16, 17]

$$\varepsilon_{\mu\nu\lambda\rho}\partial_\lambda\partial_\rho\theta^{\rm sing.}(x) = 2\pi\Sigma_{\mu\nu}(x), \tag{8}$$

and the integration measure becomes factorized, $\mathcal{D}\theta = \mathcal{D}\theta^{\rm sing.}\mathcal{D}\theta^{\rm reg.}$. Here, $\Sigma_{\mu\nu}$ stands for the vorticity tensor current, defined at the world sheet Σ of a closed dual string, parametrized by the vector $x_\mu(\xi)$. On the other hand, the field $\theta^{\rm reg.}(x)$ describes simply a singlevalued fluctuation around the above-mentioned string configuration. Note that Eq. (8) is nothing, but the Stokes' theorem for $\partial_\mu\theta^{\rm sing.}$, written in the local form.

The string representation of the theory (7) can be derived analogously to Ref. [16], where this has been done for a model with a global U(1)-symmetry. One obtains

$$\mathcal{Z} = \int \mathcal{D}x_\mu(\xi)\mathcal{D}h_{\mu\nu} \exp\left\{-\int d^4x \left[\frac{1}{24\eta^2}H^2_{\mu\nu\lambda} + \frac{g_m^2}{4}h^2_{\mu\nu} + i\pi h_{\mu\nu}\hat{\Sigma}_{\mu\nu}\right]\right\}, \tag{9}$$

where $\hat{\Sigma}_{\mu\nu} \equiv 2\Sigma^{\rm e}_{\mu\nu} - \Sigma_{\mu\nu}$, and $H_{\mu\nu\lambda} \equiv \partial_\mu h_{\nu\lambda} + \partial_\lambda h_{\mu\nu} + \partial_\nu h_{\lambda\mu}$ is the field-strength tensor of a massive antisymmetric spin-1 tensor field $h_{\mu\nu}$. This field emerged as a solution of some constraints arising from the integration over $\theta^{\rm reg.}$ and represents the massive dual vector boson. As far as the integration over the world sheets of closed strings, $\int \mathcal{D}x_\mu(\xi)$, is concerned, it appeared from the integration over $\theta^{\rm sing.}$ by virtue of Eq. (8), which established a one-to-one correspondence between $\theta^{\rm sing.}$ and $x_\mu(\xi)$. Physically this correspondence stems from the fact that the singularity of the phase of the dual Higgs field takes place just at closed-string world sheets. [Notice that, since in what follows we will be interested in effective actions, rather than the integration measures, the Jacobian emerging during the change of the integration variables $\theta^{\rm sing.} \to x_\mu(\xi)$, which has been evaluated in Ref. [18], will not be discussed below and is assumed to be included in the measure $\mathcal{D}x_\mu(\xi)$.]

Finally, the Gaussian integration over the field $h_{\mu\nu}$ in Eq. (9) leads to the following expression for the partition function (7):

$$\mathcal{Z} = \exp\left[-\frac{g^2}{2}\oint\limits_C dx_\mu \oint\limits_C dy_\mu D_m^{(4)}(x-y)\right] \times$$

$$\times \int \mathcal{D}x_\mu(\xi) \exp\left[-2(\pi\eta)^2 \int d^4x \int d^4y \hat{\Sigma}_{\mu\nu}(x) D_m^{(4)}(x-y) \hat{\Sigma}_{\mu\nu}(y)\right]. \qquad (10)$$

Here, $D_m^{(4)}(x) \equiv mK_1(m|x|)/(4\pi^2|x|)$ is the propagator of the dual vector boson, and K_ν's henceforth stand for the modified Bessel functions. Clearly, the first exponential factor on the R.H.S. of Eq. (10) is the standard result, which can be obtained without accounting for the dual Nielsen-Olesen strings. Contrary to that, the integral over string world sheets on the R.H.S. of that equation stems just from the contribution of closed strings to the partition function and is the essence of the string representation. The respective string effective action describes both the interaction of closed world sheets Σ's with the open world sheets $\Sigma^{\rm e}$'s and self-interactions of these objects.

We are now in the position to discuss the bilocal correlator of electric field strengths in the model (7). Indeed, owing to the Stokes' theorem, such an extended partition function (which is actually nothing, but the Wilson loop of a test quark) can be written as $\left\langle \exp\left(-\frac{ig}{2}\int d^4x \Sigma^{\rm e}_{\mu\nu} f_{\mu\nu}\right)\right\rangle_{a_\mu, j_\mu^{\rm m}}$, where $\langle\ldots\rangle_{a_\mu, j_\mu^{\rm m}} \equiv \left\langle \int \mathcal{D}a_\mu \exp\left(-S_{\rm eff.}\left[a_\mu, f_{\mu\nu}^{\rm m}\right]\right)(\ldots)\right\rangle_{j_\mu^{\rm m}}$ with $S_{\rm eff.}$ and $\langle\ldots\rangle_{j_\mu^{\rm m}}$ given by Eqs. (2) and (4), respectively. Applying to this expression the cumulant expansion, we have in the bilocal approximation:

$$\mathcal{Z} \simeq \exp\left[-\frac{g^2}{8}\int d^4x \int d^4y \Sigma^{\rm e}_{\mu\nu}(x)\Sigma^{\rm e}_{\lambda\rho}(y) \langle\langle f_{\mu\nu}(x) f_{\lambda\rho}(y)\rangle\rangle_{a_\mu, j_\mu^{\rm m}}\right]. \qquad (11)$$

Following the SVM, let us parametrize the bilocal cumulant $\langle\langle f_{\mu\nu}(x) f_{\lambda\rho}(0)\rangle\rangle$ similarly to the parametrization of Eq. (1), namely set for this quantity the following *Ansatz*:

$$\Big(\delta_{\mu\lambda}\delta_{\nu\rho} - \delta_{\mu\rho}\delta_{\nu\lambda}\Big)\mathcal{D}\left(x^2\right) + \frac{1}{2}\Big[\partial_\mu\Big(x_\lambda\delta_{\nu\rho} - x_\rho\delta_{\nu\lambda}\Big) + \partial_\nu\Big(x_\rho\delta_{\mu\lambda} - x_\lambda\delta_{\mu\rho}\Big)\Big]\mathcal{D}_1\left(x^2\right). \qquad (12)$$

Owing to the Stokes' theorem, Eq. (12) yields

$$\mathcal{Z} \simeq \exp\left\{-\frac{1}{8}\int d^4x \int d^4y \left[2g^2 \Sigma^{\rm e}_{\mu\nu}(x)\Sigma^{\rm e}_{\mu\nu}(y)\mathcal{D}\left((x-y)^2\right) + j^{\rm e}_\mu(x) j^{\rm e}_\mu(y) \int\limits_{(x-y)^2}^{\infty} dt \mathcal{D}_1(t)\right]\right\}. \qquad (13)$$

On the other hand, Eq. (13) should coincide with Eq. (10) divided by $\mathcal{Z}\left[\Sigma^{\rm e}_{\mu\nu}=0\right]$ (that is just the standard normalization condition, encoded in the integration measures), *i.e.* it reads

$$\mathcal{Z}=\exp\left\{-\int d^4x\int d^4yD_m^{(4)}(x-y)\left[8(\pi\eta)^2\Sigma^{\rm e}_{\mu\nu}(x)\Sigma^{\rm e}_{\mu\nu}(y)+\frac12 j^{\rm e}_\mu(x)j^{\rm e}_\mu(y)\right]\right\}\times$$

$$\times\left\langle\exp\left[8(\pi\eta)^2\int d^4x\int d^4yD_m^{(4)}(x-y)\Sigma^{\rm e}_{\mu\nu}(x)\Sigma_{\mu\nu}(y)\right]\right\rangle_{x_\mu(\xi)}, \tag{14}$$

where the average $\langle\ldots\rangle_{x_\mu(\xi)}$ is defined *w.r.t.* the action $2(\pi\eta)^2\int d^4x\int d^4y\Sigma_{\mu\nu}(x)D_m^{(4)}(x-y)\Sigma_{\mu\nu}(y)$. As it has already been discussed in the Introduction, in the semi-classical approximation, closed dual strings can be disregarded, since their typical areas $|\Sigma|$'s are much smaller than the area $|\Sigma^{\rm e}|$ of the world sheet of a long open string, which confines a test quark. Owing to this fact, the exponential factor, which should be averaged over closed strings on the R.H.S. of Eq. (14), may be disregarded *w.r.t.* the first exponential factor in that equation, as well. Then, the comparison of the latter one with Eq. (13) readily yields for the functions $\mathcal{D}$ and $\mathcal{D}_1$ the following expression

$$\mathcal{D}\left(x^2\right)=\frac{m^3}{4\pi^2}\frac{K_1(m|x|)}{|x|}, \tag{15}$$

$$\mathcal{D}_1\left(x^2\right)=\frac{m}{2\pi^2x^2}\left[\frac{K_1(m|x|)}{|x|}+\frac{m}{2}\Big(K_0(m|x|)+K_2(m|x|)\Big)\right]. \tag{16}$$

In the IR limit, $|x|\gtrsim m^{-1}$, the asymptotic behaviours of the coefficient functions (15) and (16) are given by

$$\mathcal{D}\longrightarrow\frac{m^4}{4\sqrt2\pi^{\frac32}}\frac{{\rm e}^{-m|x|}}{(m\,|x|)^{\frac32}} \tag{17}$$

and

$$\mathcal{D}_1\longrightarrow\frac{m^4}{2\sqrt2\pi^{\frac32}}\frac{{\rm e}^{-m|x|}}{(m\,|x|)^{\frac52}}. \tag{18}$$

One can now see that, according to the lattice data [3, 4], the asymptotic behaviours (17) and (18) are very similar to the IR ones of the nonperturbative parts of the functions D and D_1, which parametrize the bilocal cumulant (1) in QCD. In particular, both functions decrease exponentially, and the function $\mathcal{D}$ is much larger than the function $\mathcal{D}_1$ due to the

preexponential power-like behaviour. We also see that the role of the correlation length of the vacuum, T_g, *i.e.* the distance at which the functions D and D_1 decrease, is played in the model (7) by the inverse mass of the dual vector boson, m^{-1}.

Hence we see that, within the approximation when the contribution of closed strings to the partition function (14) is disregarded, the bilocal approximation to the SVM is an exact result in the theory (7), *i.e.* all the cumulants of the orders higher than the second one vanish. Higher cumulants naturally appear upon performing in Eq. (14) the average over closed strings. However, this average yields important modifications already on the level of the bilocal cumulant. Namely, as we will see in the next Section, it modifies the semi-classical expressions (15) and (16).

CORRECTIONS TO THE $\bar{Q}Q$-POTENTIAL PRODUCED BY CLOSED STRINGS AND A FINITE HIGGS MASS

To study the properties of closed strings, it is enough to consider the theory without external quarks. The field-strength correlators can be addressed afterwards, *i.e.* already after the summation over the grand canonical ensemble of closed strings. Thus, let us first consider the theory (7) with $F^{\rm e}_{\mu\nu} = 0$. Upon the derivation of the string representation of such a theory, we are then left with Eq. (9), where $\Sigma^{\rm e}_{\mu\nu} = 0$. To study the grand canonical ensemble of closed strings, it is necessary to replace $\Sigma_{\mu\nu}$ in Eq. (9) by the following expression: $\Sigma^N_{\mu\nu}(x) = \sum\limits_{i=1}^{N} n_i \int d\sigma_{\mu\nu}(x_i(\xi))\delta(x - x_i(\xi))$. Here, n_i's stand for winding numbers. In what follows, we will restrict ourselves to closed strings possessing the minimal winding numbers, $n_i = \pm 1$. That is because, analogously to the 3D-case [13, 19], the energy of a single closed string is known to be a quadratic function of its flux, owing to which the vacuum prefers to maintain two closed strings of a unit flux, rather than one string of the double flux.

Then, taking into account that the plasma of closed strings is dilute, one can perform the summation over the grand canonical ensemble of these objects, that yields [instead of Eq. (9)] the following expression for the partition function:

$$\mathcal{Z} = \int \mathcal{D}h_{\mu\nu} \exp\left\{-\int d^4x \left[\frac{1}{24\eta^2} H^2_{\mu\nu\lambda} + \frac{g_m^2}{4} h^2_{\mu\nu} - 2\zeta \cos\left(\frac{|h_{\mu\nu}|}{\Lambda^2}\right)\right]\right\}. \tag{19}$$

Here $|h_{\mu\nu}| \equiv \sqrt{h^2_{\mu\nu}}$, and $\Lambda \equiv \sqrt{\frac{L}{a^3}}$ is an UV momentum cutoff with L and a denoting the characteristic distances between closed strings and their typical sizes, respectively. Clearly,

in the dilute-plasma approximation under study, $a \ll L$ and $\Lambda \gg a^{-1}$. Also in Eq. (19), $\zeta \propto \mathrm{e}^{-S_0}$ stands for the fugacity (Boltzmann factor) of a single string, which has the dimension $(\text{mass})^4$, with S_0 denoting the action of a single string. The value of S_0 parametrically equals σa^2, where the area of the string world sheet is proportional to a^2, and σ is the string tension; $\sigma \simeq 2\pi\eta^2 \ln\left(\frac{\lambda}{g_m^2}\right)$ in the London limit $\ln\left(\frac{\lambda}{g_m^2}\right) \gg 1$.

The square of the full mass of the field $h_{\mu\nu}$ following from Eq. (19) reads $M^2 = m^2 + m_D^2 \equiv Q^2\eta^2$. Here, $m_D^2 = 8\zeta\eta^2/\Lambda^4$ is the additional contribution, emerging due to the Debye screening of the dual vector boson in the plasma of closed strings, and $Q^2 = 2\left(g_m^2 + \frac{4\zeta}{\Lambda^4}\right)$ is the (squared) full magnetic charge of the dual vector boson.

To study the correlation functions of closed strings, it is convenient to represent the partition function (19) directly as an integral over the densities of these objects. This can be done by means of some kind of a Legendre transformation, and the resulting action reads

$$S = 2(\pi\eta)^2 \int d^4x \int d^4y \Sigma_{\mu\nu}(x) D_m^{(4)}(x-y)\Sigma_{\mu\nu}(y) + V[\Sigma_{\mu\nu}], \tag{20}$$

where the effective potential of closed strings, V, is

$$V[\Sigma_{\mu\nu}] = \int d^4x \left\{\Lambda^2|\Sigma_{\mu\nu}| \ln\left[\frac{\Lambda^2}{2\zeta}|\Sigma_{\mu\nu}| + \sqrt{1+\left(\frac{\Lambda^2}{2\zeta}|\Sigma_{\mu\nu}|\right)^2}\right] - 2\zeta\sqrt{1+\left(\frac{\Lambda^2}{2\zeta}|\Sigma_{\mu\nu}|\right)^2}\right\}. \tag{21}$$

It can be proved that the correlation functions of $\Sigma_{\mu\nu}$'s, evaluated by virtue of the representation (20), are nothing, but the correlation functions of densities of closed strings in the plasma. These correlation functions can be calculated in the approximation when the plasma is sufficiently dilute, namely its density obeys the inequality $|\Sigma_{\mu\nu}| \ll \frac{\zeta}{\Lambda^2}$, and the potential (21) becomes a simple quadratic functional of $\Sigma_{\mu\nu}$'s. In particular, the simplest nontrivial correlation function $\left\langle\left\langle\Sigma_{\mu\nu}(y)\Sigma_{\lambda\rho}(y')\right\rangle\right\rangle_{x_\mu(\xi)}$ can be evaluated in this approximation. Inserting further the so-obtained expression for this correlation function into the average on the R.H.S. of Eq. (14) (evaluated by means of the cumulant expansion in the bilocal approximation), one obtains for the functions $\mathcal{D}$ and $\mathcal{D}_1$ [12]:

$$\mathcal{D}^{\text{full}}\left(x^2\right) = \frac{m^2 M}{4\pi^2}\frac{K_1(M|x|)}{|x|}, \tag{22}$$

$$\mathcal{D}_1^{\rm full}\left(x^2\right) = \frac{m_D^2}{\pi^2 M^2 |x|^4} + \frac{m^2}{2\pi^2 M x^2}\left[\frac{K_1(M|x|)}{|x|} + \frac{M}{2}\left(K_0(M|x|) + K_2(M|x|)\right)\right]. \qquad (23)$$

We see that, as it should be, the functions (22) and (23) go over into Eqs. (15) and (16), respectively, when $m_D \to 0$, *i.e.* when one neglects the effect of screening in the ensemble of closed strings. An obvious important consequence of the obtained Eqs. (22) and (23) is that the correlation length of the vacuum, T_g, becomes modified from m^{-1} [according to Eqs. (15) and (16)] to M^{-1}. (It is worth pointing out once again that this effect is due to the Debye screening of the dual vector boson in the ensemble of closed strings, that makes this particle heavier, namely its mass becomes increased from m to M.) Indeed, it is straightforward to see that, at $|x| \gtrsim M^{-1}$,

$$\mathcal{D}^{\rm full} \longrightarrow \frac{(mM)^2}{4\sqrt{2}\pi^{\frac32}} \frac{{\rm e}^{-M|x|}}{(M|x|)^{\frac32}}, \quad \mathcal{D}_1^{\rm full} \longrightarrow \frac{m_D^2}{\pi^2 M^2 |x|^4} + \frac{(mM)^2}{2\sqrt{2}\pi^{\frac32}} \frac{{\rm e}^{-M|x|}}{(M|x|)^{\frac52}}.$$

A remarkable fact is that the leading term of the IR asymptotics of the function $\mathcal{D}_1^{\rm full}$ is a pure power-like one, rather than that of the function $\mathcal{D}_1$, given by Eq. (18). This term produces a nonperturbative $(1/r)$-contribution to the $\bar{q}q$-potential, $\Delta V(r) = -\frac{(m_D/M)^2}{4\pi r}$, which by its structure resembles the Lüscher term. Typically, modelling the Lüscher term within the SVM is rather problematic. Indeed, in the standard approach, in order to get the Lüscher term, one should consider string fluctuations, while SVM is well defined only on the minimal-area surface (see *e.g.* Ref. [2]). Now, we have found another mechanism, which might generate a Lüscher-type term via a novel nonperturbative perimeter interaction.

It is also worth noting that, despite the modification of the $\mathcal{D}$-function, the string tension of the open dual-string world sheet $\Sigma^{\rm e}$, $\sigma = 4T_g^2 \int d^2z \mathcal{D}\left(z^2\right)$ (*cf.* Ref. [20]), becomes modified only by means of the logarithm of the Landau-Ginzburg parameter. Indeed, one obtains $\sigma = 8\pi\eta^2 \ln(\lambda/Q^2) \propto \eta^2$, and η is not affected by the Debye screening. The screening rather modifies more significantly the coupling constant of the next-to-leading term in the derivative expansion of the nonlocal string effective action (the so-called rigidity term). Indeed, by virtue of the results of Ref. [20], one can see that, for the same world sheet $\Sigma^{\rm e}$, this coupling constant without taking screening into account reads $-\frac{\pi}{2g_m^2}$, whereas in the presence of screening it goes over to $-\frac{\pi}{2\left(g_m^2 + \frac{4\zeta}{\Lambda^4}\right)} = -\frac{\pi}{Q^2}$.

Another origin of corrections to the $\bar{q}q$-potential (even without accounting for closed strings) is due to the deviation from the London limit [21]:

$$V(r) = -g^2 \frac{\mathrm{e}^{-mr}}{4\pi r} \left[1 - \mathrm{e}^{-\left(\sqrt{m^2+m_H^2}-m\right)r} + \mathrm{e}^{-(m_H-m)r} \right], r > m_H^{-1}.$$

Clearly, this potential is neither Yukawa, nor Coulombic one, but it goes to the Yukawa potential in the London limit $m_H \to \infty$.

STRING REPRESENTATION OF THE SU(N)-INSPIRED DAHM WITH THE Θ-TERM

In this Section, we will present a string representation of the SU(N)-inspired analogue of the model (6), extended, for completeness, by the Θ-term. Owing to this term, quarks acquire a nonvanishing magnetic charge (*i.e.*, become dyons) and scatter off closed dual strings. As one of the consequences of our result, we will get the critical values of Θ, at which the long-range topological interaction of dual strings with dyons disappears. These values, in particular, reproduce the respective SU(2)- and SU(3)-ones, found in Refs. [22] and [23], respectively. The partition function of the effective $[U(1)]^{N-1}$ gauge-invariant Abelian-projected theory we are going to explore reads

$$\mathcal{Z}_\alpha = \int \left(\prod_i |\Phi_i| \mathcal{D} |\Phi_i| \mathcal{D}\theta_i \right) \mathcal{D}\mathbf{B}_\mu \delta \left(\sum_i \theta_i \right) \exp\left\{ - \int d^4x \left[\frac{1}{4} \left(\mathbf{F}_{\mu\nu} + \mathbf{F}_{\mu\nu}^{(\alpha)} \right)^2 + \right.\right.$$

$$\left.\left. + \sum_i \left[|(\partial_\mu - ig_m \mathbf{q}_i \mathbf{B}_\mu) \Phi_i|^2 + \lambda \left(|\Phi_i|^2 - \eta^2 \right)^2 \right] - \frac{i\Theta g_m^2}{16\pi^2} \left(\mathbf{F}_{\mu\nu} + \mathbf{F}_{\mu\nu}^{(\alpha)} \right) \left(\tilde{\mathbf{F}}_{\mu\nu} + \tilde{\mathbf{F}}_{\mu\nu}^{(\alpha)} \right) \right] \right\}. \quad (24)$$

Here, the index i runs from 1 to the number of positive roots $\mathbf{q}_i$'s of the SU(N)-group, that is $N(N-1)/2$. Note that the origin of root vectors in Eq. (24) is the fact that monopole charges are distributed along them. Further, $\Phi_i = |\Phi_i| \mathrm{e}^{i\theta_i}$ are the dual Higgs fields, which describe the condensates of monopoles, and $\mathbf{F}_{\mu\nu} = \partial_\mu \mathbf{B}_\nu - \partial_\nu \mathbf{B}_\mu$ is the field-strength tensor of the $(N-1)$-component "magnetic" potential $\mathbf{B}_\mu$. The latter is dual to the "electric" potential, whose components are diagonal gluons. Since the SU(N)-group is special, the phases θ_i's of the dual Higgs fields are related to each other by the constraint $\sum\limits_i \theta_i = 0$, which is imposed by introducing the corresponding δ-function into the R.H.S. of Eq. (24) [*cf.* Ref. [7] for the SU(3)-case]. Next, the index α runs from 1 to N and denotes a certain quark colour. Finally, $\mathbf{F}_{\mu\nu}^{(\alpha)}$ is the field-strength tensor of a test quark of the colour α, which

moves along a certain contour C. This tensor obeys the equation $\partial_\mu \tilde{\mathbf{F}}^{(\alpha)}_{\mu\nu} = g\mathbf{m}_\alpha j_\nu$, where $j_\mu(x) = \oint\limits_C dx_\mu(\tau)\delta(x - x(\tau))$, and $\mathbf{m}_\alpha$ is a weight vector of the fundamental representation of the group SU(N). One thus has $\mathbf{F}^{(\alpha)}_{\mu\nu} = -g\mathbf{m}_\alpha \tilde{\Sigma}^{\rm e}_{\mu\nu}$. Note further that the Θ-term can be rewritten as

$$-\frac{i\Theta g_m^2}{16\pi^2}\left(\mathbf{F}_{\mu\nu} + \mathbf{F}^{(\alpha)}_{\mu\nu}\right)\left(\tilde{\mathbf{F}}_{\mu\nu} + \tilde{\mathbf{F}}^{(\alpha)}_{\mu\nu}\right) = \frac{i\Theta g_m}{\pi}\mathbf{m}_\alpha \int d^4x \mathbf{B}_\mu j_\mu. \tag{25}$$

This means that, by means of the Θ-term, quarks acquire a nonvanishing magnetic charge $\Theta g_m/\pi$, *i.e.* become dyons, that enables them to interact with the magnetic gauge field $\mathbf{B}_\mu$ [24].

Expanding for a while $|\Phi_i|$ around the Higgs *v.e.v.* η, one gets the mass of the dual vector boson, $m = g_m\eta\sqrt{N}$. In what follows, we will again consider the London limit of the model (24), which admits a construction of the string representation. This is the limit when m is much smaller than the mass of any of the Higgs fields, $m_H = 2\eta\sqrt{\lambda}$. Since we would like our model to be consistent with QCD, we must have $g = \sqrt{\bar{\lambda}/N}$, where $\bar{\lambda}$ remains finite in the large-N limit. Therefore, in the London limit, the Higgs coupling λ should grow with N faster than $\mathcal{O}(N^2)$, namely it should obey the inequality $\lambda \gg (2\pi N)^2/\bar{\lambda}$.

Integrating then $|\Phi_i|$'s out, we arrive at the following expression for the partition function (24) in the London limit:

$$\mathcal{Z}_\alpha = \int \left(\prod_i \mathcal{D}\theta_i^{\rm sing.}\mathcal{D}\theta_i^{\rm reg.}\right)\mathcal{D}\mathbf{B}_\mu \mathcal{D}k\delta\left(\sum_i \theta_i^{\rm sing.}\right)\exp\left\{-\int d^4x\left[\frac{1}{4}\left(\mathbf{F}_{\mu\nu} + \mathbf{F}^{(\alpha)}_{\mu\nu}\right)^2 + \right.\right.$$

$$\left.\left. + \eta^2\sum_i\left(\partial_\mu\theta_i - g_m\mathbf{q}_i\mathbf{B}_\mu\right)^2 - ik\sum_i \theta_i^{\rm reg.} - \frac{i\Theta g_m^2}{16\pi^2}\left(\mathbf{F}_{\mu\nu} + \mathbf{F}^{(\alpha)}_{\mu\nu}\right)\left(\tilde{\mathbf{F}}_{\mu\nu} + \tilde{\mathbf{F}}^{(\alpha)}_{\mu\nu}\right)\right]\right\}. \tag{26}$$

The multivalued fields $\theta_i^{\rm sing.}$'s here are related to the world sheets of closed dual strings Σ_i's by the same Eq. (8). The string representation of this partition function reads [25]

$$\mathcal{Z}_\alpha = \exp\left\{-\frac{N-1}{4N}\left[g^2 + \left(\frac{\Theta g_m}{\pi}\right)^2\right]\int d^4x d^4y j_\mu(x) D_m(x-y) j_\mu(y)\right\}\int\left(\prod_i \mathcal{D}x^{(i)}(\xi)\right)\times$$

$$\times\delta\left(\sum_i \Sigma^i_{\mu\nu}\right)\exp\left[-2(\pi\eta)^2\int d^4x d^4y \hat{\Sigma}^i_{\mu\nu}(x) D_m(x-y)\hat{\Sigma}^i_{\mu\nu}(y) - 2i\Theta s_i^{(\alpha)}\hat{L}\left(\Sigma_i, C\right) + \right.$$

$$+ 2i\Theta \int d^4xd^4y \left(\frac{N-1}{N}\tilde{\Sigma}_{\mu\nu}(x) - s_i^{(\alpha)}\tilde{\Sigma}^i_{\mu\nu}(x)\right) j_\mu(y)\partial^x_\nu D_m(x-y)\Bigg], \tag{27}$$

where $\hat{\Sigma}^i_{\mu\nu} \equiv \Sigma^i_{\mu\nu} - Ns_i^{(\alpha)}\Sigma_{\mu\nu}$, and nonvanishing $s_i^{(\alpha)}$'s are equal $\pm N^{-1}$. Note that, for every color α, it is straightforward to integrate out one of the world sheets Σ_i's by resolving the constraint imposed by the δ-function.

The first exponent on the R.H.S. of Eq. (27) represents the short-ranged interaction of quarks via dual vector bosons. Noting that, for any α, $\mathbf{m}_\alpha^2 = (N-1)/(2N)$, one readily deduces from this term the total charge of the quark, $\sqrt{g^2 + (\Theta g_m/\pi)^2}$. The magnetic part of this charge coincides with the one stemming from Eq. (25). Further, the first term in the second exponent on the R.H.S. of Eq. (27) is again the short-ranged (self-)interaction of closed world sheets Σ_i's and an open one Σ, responsible for confinement. The last term on the R.H.S. of Eq. (27) describes the short-range interactions of dyons with both closed and open strings (obviously, the latter confine these very dyons themselves). Instead, the term $-2i\Theta s_i^{(\alpha)}\hat{L}\left(\Sigma_i, C\right)$ in Eq. (27) describes the long-range interaction of dyons with closed world sheets, that is the 4D-analogue of the Aharonov-Bohm effect [26]. Since nonvanishing values of $s_i^{(\alpha)}$'s are equal $\pm N^{-1}$, at $\Theta \neq N\pi \times$ integer, dyons (due to their magnetic charge) do interact by means of this term with the closed dual strings. On the contrary, these critical values of Θ correspond to such a relation between the magnetic charge of a dyon and an electric flux inside the string when the scattering of dyons off strings is absent.

SUMMARY

In the present article, we have first briefly reviewed the properties of electric field-strength correlators in the DAHM, which correspond to the gauge-invariant correlators in the real QCD. First, we have reviewed the semi-classical analysis of these correlators. Then, the leading correction to this result, produced by the interaction of the open-string world sheet with closed dual strings, has been evaluated. This effect is essentially quantum, as well as the plasma of closed strings itself. In this way, it has been shown that the correlation length of the vacuum becomes modified from the inverse mass of the dual vector boson, which it acquires by means of the Higgs mechanism, to its inverse full mass, which takes into account also the effect of Debye screening. What is more important is that, in one of the two coefficient functions, which parametrize the bilocal correlator of electric field strengths

within the SVM, a nonperturbative power-like IR part appears, which was absent on the semi-classical level. This novel term opens up a possibility of generating a Lüscher-type term within the SVM. We have further presented another type of modification of the $\bar{q}q$-potential, which appears beyond the London limit. The novel potential is a certain combination of Yukawa potentials with various effective masses, but it goes over to the standard Yukawa potential in the London limit. Finally, we have discussed the string representation of the SU(N)-counterpart of DAHM in the London limit, extended by the Θ-term. Owing to the latter, quarks have been shown to acquire a magnetic charge and scatter off closed dual strings, provided Θ does not take its values from a certain discrete set.

In conclusion, the obtained results demonstrate similarities in the vacuum structures of DAHM and QCD by means of the SVM. They might also shed some light on the origin of the Lüscher term in QCD, as well as on the structure of the colour flux tubes.

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